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Controlling Decoherence from Fluctuating Magnetic Field

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Abstract A different strategy is proposed to control decoherence from fluctuating magnetic field by adjusting external controllable parameters. The results show that the output states in terms of the fidelity are pure states, which correspond to the state vectors that are given by a renormalized Hamiltonian. Thus, the output states may perfectly preserve memory of initial single-qubit states at some critical magnetic field parameters.

Keywords Decoherence · Controllable parameter · Fluctuating magnetic field

In a really physical word, the interaction between a quantum system with its surrounding environment may lead to an irreversible loss of information on the system. Because the interacting effect is that quantum superpositions decay into statistical mixtures so as to result in a relatively short coherence time, the decoherent process limits the ability to maintain pure quantum states in quantum information processing [1–3]. Thus, noise and decoherence are a major challenge how to preserve quantum coherent state in practical applications [4–9].

Several schemes have been proposed to solve the problem, which included quantum error correction strategies [10–13], feedback implementations [14–16], the realization of qubits in symmetric subspaces decoupled from the environment [17–19], dynamical decoupling techniques [20, 21], and engineering of pointer states [22]. Though the engineering of pointer states and feedback implementations were proposed to maintain a single-qubit

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state, the engineering approach may lead to new decoherent sources because of the interaction between artificial reservoirs and qubit, while the singularities are a well-known feature of feedback implementations by tracking control [23–26]. Moreover, the others are inadequate to be applied to a single-qubit open system, where both quantum error correcting codes and decoherence-free subspaces are dependent on an encoding of single-into several-qubit states and the dynamical decoupling is inapplicable in a fully decoherent regime.

It is known that the decoherence may come from the fluctuations of both physical quantities and vacuums. In previous works, however, almost all studies have concentrated to protect against the destructive effects of vacuum fluctuations on single- or many-qubit system. Therefore, it is worth investigating how to control the effects of fluctuating magnetic field in quantum information. It may be very important, especially, for process of quantum information because noisy mixing of matter would also mimic a fluctuating matter [27, 28].

In this paper, a simply physical strategy is proposed to overcome the decoherence, where a given single-qubit state is protected against the destructive effects of interaction between the fluctuating magnetic field and two-level atomic system. By controlling some external parameters of physical system to satisfy a transcendental equation with the fluctuating rate, one may obtain an output of quantum pure state. Furthermore, by choosing evolving time, we may protect an initial quantum state with a perfect fidelity.

Let us consider a two-level atom in the presence of an external magnetic field. The Hamiltonian of system is described by $H_0 = \frac{1}{2}\hbar\Omega\sigma_z$, where σ_i $(i = x, y, z)$ are the Pauli operators and $\Omega = g\mu B/\hbar$ with $g(\mu)$ are the gyromagnetic, B acts as an external controllable parameter and can be experimentally changed. We further extend our analysis to contain a fluctuating component. In this case a master equation can be written for the atomic density operator ρ in the Schrödinger picture,

$$i\hbar\frac{d}{dt}\rho(t) = [H(t), \rho(t)], \tag{1}$$

where the Hamiltonian is taken to be linear in a fluctuating field, i.e.,

$$H(t) = H_0 + B(t)M, \tag{2}$$

where $B(t)$ is a random field and $M = \frac{1}{2}\hbar\sigma_x$ is independent of time. It is obvious that the random field is a decoherent source in our system. After averaging on different trajectories induced by the noise, actually, the system at the end of the evolution is in a mixed state under the case without controlling magnetic field. Thus the Bloch vector, described the motion evolution of mixed state, does not return to its initial position since the Hamiltonian does not. To calculate the correct Bloch vector, we have to average the final positions [29]. Thus, the master equation (1) is rewritten as

$$\frac{d}{dt}\langle\rho(t)\rangle = -\frac{i}{\hbar}[H_0, \langle\rho(t)\rangle] - \frac{k(t)}{2\hbar^2}[M(t), [M(t), \langle\rho(t)\rangle]], \tag{3}$$

where $\langle\cdots\rangle$ represents the average over the random variables with the average value $k(t)$. The double commutator is a violating term of the charge conjugation, parity and time reversal symmetry (CPT) since, although it is CP symmetric, it induces time-irreversibility, which is as an appropriate form for the decoherence. It is noted that (3) is different from the others from a system interacting with the normalized [2, 3, 29] and squeezed vacuum fluctuation [30].

It is known that the density matrix was introduced as a way of describing a quantum open system and the state of the open system is not completely known. In a general case, a state for the open system can always be written in many different ways as a probabilistic mixture of distinct but not necessarily orthogonal pure states.

In general case, the random fluctuation of the magnetic field is small in comparison with the atom oscillation length. Therefore, we only consider the case of $\Omega_r^2 = \Omega^2 - \kappa^2 \geq 0$, where $\kappa = k/4$ will be taken as a slow change function to time.

The solution of master equation (3) is direct. The diagonal elements of the density matrix can be expressed by

$$\langle\rho_{11}\rangle = \frac{1}{2}\left(1 + \cos\theta e^{-(2\Omega\cos\phi)t}\right), \qquad \langle\rho_{22}\rangle = 1 - \langle\rho_{11}\rangle, \tag{4}$$

where $\phi = \arctan(\Omega_r/\kappa)$. From (4), we see that the diagonal elements of the density matrix include an effect of decay, which is similar to the spontaneous decay of an atom interacting with the normal-vacuum reservoir.

For the nondiagonal elements of the density matrix, one finds

$$\begin{aligned}\langle\rho_{12}\rangle &= \frac{1}{4}\frac{\sin\theta}{\sin\phi}e^{-\Omega\cos\phi t}[(-1 + ie^{-i\phi})e^{-i\Omega\sin\phi t} + (1 - ie^{i\phi})e^{i\Omega\sin\phi t}],\\ \langle\rho_{21}\rangle &= \langle\rho_{12}\rangle^*.\end{aligned} \tag{5}$$

From (5), we see that both the complex oscillations and exponential decay with the evolving time will be included in the nondiagonal elements of density matrix. When the fluctuating parameter $\kappa = 0$, the oscillations become simple with the frequency Ω related to the magnetic field. Here the decoherent factor $e^{-\Omega\cos\phi t}$ parameterizes the amount of decoherence, which is a similar effect of dephasing to decrease the size of the nondiagonal elements of the density matrix in a basis determined by the fluctuating interaction with the environment so that a single-qubit is corrupted by the fluctuation.

The properties of quantum information through noisy quantum channels are quantified by the fidelity which measures the overlap between the initial and time-developed state vectors [31–33]. For an initially pure state $|\psi(0)\rangle$, the fidelity is in fact a probability to find the initial state in the output state at a later time. Now we take the density matrix $\langle\rho(0)\rangle = |\psi(0)\rangle\langle\psi(0)|$ as an initial state at the time $t = 0$ and $\langle\rho(t)\rangle$ in (6)–(9) as a finial state at the time $t = \tau$, respectively. The fidelity of two-level atomic system with the fluctuating field is written as

$$\begin{aligned}F(\tau) &= \mathrm{Tr}\left(\langle\rho(0)\rangle\langle\rho(\tau)\rangle\right)\\ &= \frac{1}{2} + \frac{1}{2}\cos^2\theta e^{-(2\Omega\cos\phi)\tau} + \frac{1}{2}\frac{\sin^2\theta}{\sin\phi}e^{-(\Omega\cos\phi)\tau}\sin(\phi + \Omega\tau\sin\phi).\end{aligned} \tag{6}$$

From (6), we find that an similarly optical nutation is taken place in the fidelity, which is oscillated in terms of the function $\sin(\phi + \Omega\tau\sin\phi)$ as well as exponential decay in terms of the damping factors. In general, therefore, $0 \leq F(\tau) \leq 1$. For the case of $F(\tau) = 0$, the initial quantum information is completely lost in the quantum information processing. In the case of $F(\tau) = 1$, the quantum state is perfectly preserved in process of the information. If $0 < F(\tau) < 1$, the part messages are lost in the quantum information processing so that the output state preserves only a part of memory about the initial state.

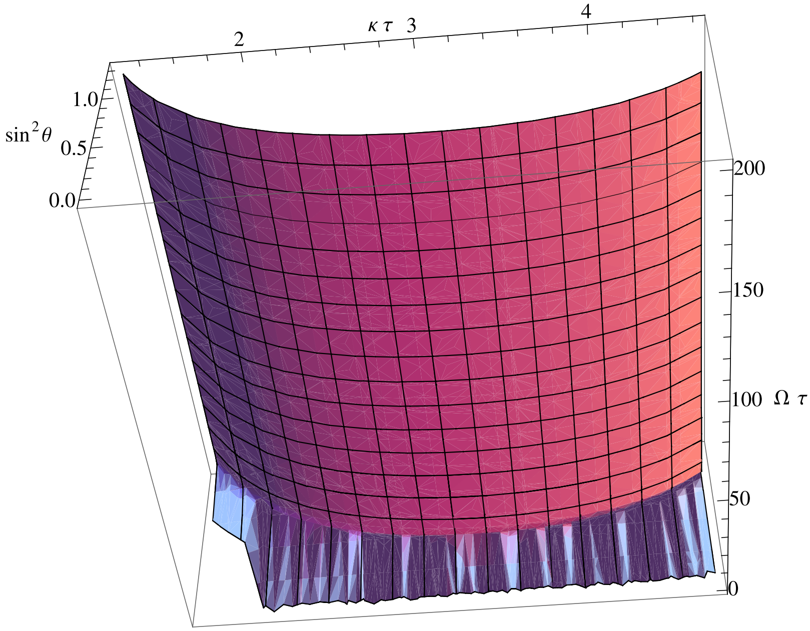

Fig. 1 Solution of transcendental equation (7). The results show that there exist some physical meaning solutions at the range $\kappa\tau \in [1.3, 4.6]$, which imply that one can obtain an output of pure state by controlling the magnetic field

Our objective is to have the decoherent factor disappear during the evolution. Thus we seek the solution of following transcendental equation,

$$\begin{aligned} &\frac{1}{2} + \frac{1}{2}\cos^2\theta e^{-(2\Omega\cos\phi)\tau} + \frac{1}{2}\frac{\sin^2\theta}{\sin\phi}e^{-(\Omega\cos\phi)\tau}\sin(\phi + \Omega\tau\sin\phi) \\ &\quad = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\Omega_t\tau), \end{aligned} \tag{7}$$

where $\Omega_t = \Omega\sin\phi = \Omega_r$. It is worth noting that the conditional frequency Ω_t satisfied (7) is dependent on the fluctuating rate and initial angle of single-qubit state and may be controlled by the experimenters. We find that there exist, indeed, some physically meaningful solutions in (7) for many regions so that the scheme may be realized to overcome the fluctuating decoherence. The numerical results for the solution of conditional frequency $\Omega\tau$ as the functions of $\sin^2\theta$ and $\kappa\tau$ are shown at Figs. 1–2 for the regions $\kappa\tau \in [1.3, 5]$ and $\kappa\tau \in [64.4, 67.5]$, respectively. At Figs. 1–2, the results show that, for the fluctuating rate κ in the two regions, there exists a corresponding frequency $\Omega(\theta, \kappa)$ satisfied (7). Comparing Fig. 1 with Fig. 2, we see that $\Omega(\theta, \kappa)$ in the region $\theta \approx 0$ is strongly dependent on the fluctuating rate. We find that, furthermore, the solution of transcendental equation (7) may be obtained in all different regions of initial angle θ and fluctuating rate κ. Similarly to Figs. 1–2, the conditional frequencies Ω in the different regions are all dependent on the fluctuating rate and initial angle of qubit. These imply that the some critical magnetic field parameters may be found to control the decoherence from fluctuating magnetic field in all regions.

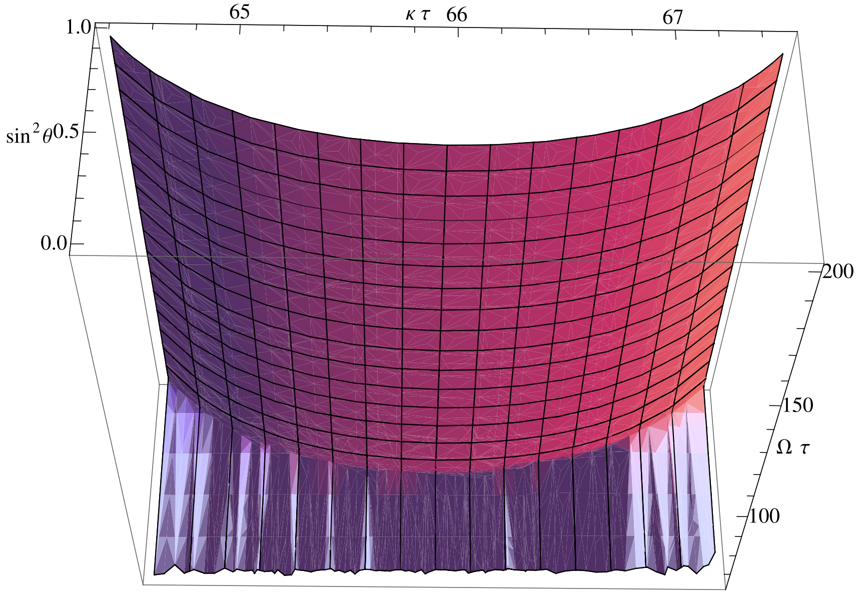

Fig. 2 Same for the range $\kappa\tau \in [64.4, 67.5]$

Inserting (7) into (6), the fidelity becomes

$$F(\tau) = \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2} + \frac{1}{2}\sin^2\theta\cos(\Omega_t\tau). \tag{8}$$

From (8), we see that, though the fidelity depends on the fluctuating rate in terms of the conditional frequency $\Omega_t(\theta, \kappa)$, the decoherent factors disappear.

It is obvious that the corresponding output state from (8) may be described by

$$\langle\rho(\tau)\rangle = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}\sin\theta e^{-i\Omega_t\tau} \\ \frac{1}{2}\sin\theta e^{i\Omega_t\tau} & \sin^2(\theta/2) \end{pmatrix}. \tag{9}$$

It is well-known that the density matrix of two-level atom can be expressed by a Bloch sphere, such as $\langle\rho\rangle = \frac{1}{2}(1 + \mathbf{n}\cdot\overrightarrow{\sigma})$, where $\mathbf{n}$ may be parameterized as a Bloch vector along the three pseudospin directions described by two azimuthal angles, such as α and β. Thus, we have

$$\mathbf{n} = \mathrm{Tr}\,(\langle\rho\rangle\overrightarrow{\sigma}) = (r\sin\alpha\cos\beta, r\sin\alpha\sin\beta, r\cos\alpha), \tag{10}$$

which satisfies the following relations, $\mathbf{n}^* = \mathbf{n}$, and $\mathbf{n}\cdot\mathbf{n} = r^2 \leq 1$, where r is radius of the Bloch sphere.

For $r = 1$, the physical system corresponds to a pure state and the given points on the unit Bloch sphere can be mapped onto field amplitudes as a pure state $|\psi\rangle$, where $|\psi\rangle$ is a unit vector in the complex projective Hilbert space [34–36]. In the case of $r < 1$, the physical system corresponds to a mixed state. In other words, the interior points in the unit Bloch sphere are one-to-one correspondence to mixed states [37, 38].

For the physical system described by the density matrix (9), it is easy to find that $r = 1$, $\alpha = \theta$ and $\beta = \Omega_t \tau$, which imply that the output state is a pure state. Thus, the amplitudes of wave functions may be mapped onto given points on the unit Bloch sphere by

$$n_i = \langle \psi | \sigma_i | \psi \rangle, \quad i = x, y, z. \tag{11}$$

By solving (11), a map of one-to-one correspondence for the density matrix (9) to the state vector $\psi(\tau)$ is founded by

$$|\psi(\tau)\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\Omega_\tau \tau} \sin(\theta/2) \end{pmatrix}. \tag{12}$$

Thus, an effective scheme is obtained for the perfect recovery of a single-qubit pure state. Especially, if the output state is controlled at the points of evolving times $\tau = 2n\pi/\Omega_t$ $(n = 1, 2, \ldots)$, one may obtain a perfect fidelity $F(\tau) = 1$ according to (8). This implies that a single-qubit state is completely preserved in our approach. In other words, by controlling the external magnetic filed according to (7), one may effectively overcome the decoherence for the output state in the presence of fluctuating magnetic field.

In conclusion, a way is proposed to avoid the decoherence for open system by controlling the magnetic field. Our approach is applied to analyze the two-level system interacting with a fluctuating magnetic field. The results show that, by controlling the external parameters to satisfy some relations with the decoherent rates, the output state may be a pure state. Especially, if some evolving time points $\tau = 2n\pi/\Omega_t$ are chosen for the output state, where the conditional frequency Ω_t has a shift related to the two-level system resonance frequency, one may get a perfect fidelity so that an initial qubit state is completely preserved in the output state. Therefore, it is very helpful for quantum information processing and coherent controlling.

From (9) and (12), we see that the pure states of output may be described by the Hamiltonian $H_R = \frac{1}{2}\Omega_t \hbar \sigma_z$. Therefore, our approach to avoid the decoherence may be obtained by renormalizing for the free Hamiltonian of two-level system including only the magnetic field, where the magnetic field frequency Ω is replaced by the conditional frequencies Ω_t included the fluctuating rate, respectively.

In comparison with the approaches suppressed the decoherence by the artificial reservoirs, our strategy does not need any such process, which leads to a possible reduction in experimental errors as well as in decoherent sources. In contrast to the feedback of implementations, we may completely preserve a qubit state with the perfect fidelity in the output state.

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